AN EFFICIENT FUNCTION

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Let's consider the function $F(b ; n) = b ! / (b – n) ! b^n$

This function gives the probability to have, in base b and in n occurrences, none repetition.

Here are three examples of application :

1. Let's take four consecutive digits out of a random number. The probability that these four digits have four different values is given by F (10 ; 4) = 504/1000. So there is a probability 496/1000 (nearly 0,5) that one digit is present minimum twice.

2. In a throw of five dices, what is the probability to get a sequence (a sequence is 1, 2, 3, 4, 5 or 2, 3, 4, 5, 6) ? The probability to get five different values is given by F(6 ; 5) = 5/54. We have a sequence when the missing digit is either 1 or 6, and that happens one time out of three. The probability of a sequence is 5/162 (about 0,03).

3. The « birthdays paradox »:
n persons are in a room (we assume that the days of birthday are uniformly distributed on the year, and February 29th is not taken into account.)
What is the minimum value of n for which the probability that minimum two people have the same day of birthday (example: January 7th – we don't take the year into account) is higher than 0.5?
The probability to have n different dates is F (365 ; n).
F(365 ; 23) = 0,493. For n > = 23, the probability to have at least one repetition is > ½, and the answer is 23.